Unification of Higgs and Maxwell fields in Brane-Kaluza-Klein gravity

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Abstract

The unification of Higgs and electromagnetic fields in the context of higher dimensional gravity is studied. We show that these fields arise from an extra large dimension together with a compact small dimension. The question of the localization of the gauge fields and their relation to junction conditions is also addressed.

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1 Introduction

One of the important, but unanswered questions of the physics of elementary particles relates to the origin of the spectrum of the known particles. In particular, the observed hierarchy of particle masses still suffers from the absence of a coherent and fundamental understanding. Promising ideas in this regard for the past decades have been mainly based on symmetry principles. The idea of spontaneous symmetry breaking embodied in the appearance of the Higgs field is the fruit of such attempts from which the contemporary physics has been greatly benefited. The Higgs field therefore, has been playing an undeniably important role in the realm of particle physics and cosmology ever since the idea came into fruition in the 1960's. Because of its success, people have been trying to devise mechanisms in order to give a plausible explanation for its origin.

Generally speaking, in quantum field theory, the potential representing the Higgs particle enters the theory in a rather spurious way and its origin is therefore not well explained. However, in recent years, theories of gravity with extra dimensions have become an important tool, both in particle physics and cosmology and been used in order to gain more understanding in the nature and origin of fundamental particles and forces. For example, extra compact dimensions have been used to address the hierarchy problem [1]; the huge disparity in size between various fundamental constants in nature. In another approach, large extra dimensions [2] have been used to reduce the unification scale from Plankian to that of Tev. Interestingly, these theories have also been used in the past few years to account for the origin of the Higgs field. Indeed, the Higgs field is studied by considering the extra dimensional components of higher dimensional gauge fields in five dimensions in [3], or the Higgs hierarchy problem is dealt with in [4] by considering the transverse (submillimeter) and longitudinal (Tev) extra dimensions.

In this paper, we study the Higgs field in a theory with two extra dimensions, one being compact and the other large. For the compact dimension the theory is of the Kaluza-Klien type and for the large extra dimension it follows the footsteps of the brane theory. The compact extra dimension is characterized by the gauge fields A_μ and the scalar field ϕ. A perturbation of the coordinates of the bulk space in terms of the coordinates of the brane and the normal direction together with Gauss-Codazzi equations and the imposition of the Israel junction conditions culminates in an effective action for the theory in which the potential turns out to become that of the Higgs having a mass characterized by the constants of the theory. It is interesting to note that we have made no *apriori* assumptions about the fields and their properties. The characteristic of these fields have been deduced from purely geometrical considerations.

2 Geometrical setup

Consider the background manifold $\overline{V}_5$ isometrically embedded in V_6 by the map $\mathcal{Y}:\overline{V}_5\rightarrow V_6$ such that [1]

$$\mathcal{G}_{AB}\mathcal{Y}^A_{,\mu}\mathcal{Y}^B_{,\nu}=\bar{g}_{\mu\nu},\qquad \mathcal{G}_{AB}\mathcal{Y}^A_{,\mu}\mathcal{N}^B=0,\qquad \mathcal{G}_{AB}\mathcal{N}^A\mathcal{N}^B=\epsilon,\qquad \epsilon=\pm1$$

where $\mathcal{G}_{AB}$ ($\overline{g}_{\mu\nu}$) is the metric of the bulk (brane) space V_6 ($\overline{V}_5$), $\{\mathcal{Y}^A\}$ and $\{x^\mu\}$ are the bases expanding the bulk (brane) and $\mathcal{N}^A$ is a normal unit vector orthogonal to the brane. In this paper

[1] Capital Latin indices run from 0 to 5, small Latin indices run from 0 to 3 and Greek indices run from 0 to 4. Quantities representing the bulk are distinguished from that of the brane by calligraphic letters.

we adapt the Kaluza-Klein metric as the background metric and write it as

$$\bar{g}_{\mu\nu} = \begin{pmatrix} \bar{g}^{(0)}_{ij}(x^l) + \phi^2 A_i A_j & \phi^2 A_j \\ \phi^2 A_i & \phi^2 \end{pmatrix} \tag{1}$$

where $\bar{g}^{(0)}_{ij}$ is the metric of the $4D$ spacetime , A_i is the Maxwell gauge field and ϕ is the scale factor of the small compact dimension. The integrability condition for the background geometry are given by the Gauss-Codazzi equations which can be written as [5]

$$\bar{R}_{\mu\nu\alpha\beta} = 2\bar{K}_{\mu[\beta}\bar{K}_{\alpha]\nu} + \mathcal{R}_{ABCD}\mathcal{Y}^A_{,\mu}\mathcal{Y}^B_{,\nu}\mathcal{Y}^C_{,\alpha}\mathcal{Y}^D_{,\beta} \tag{2}$$

and

$$\bar{K}_{\alpha[\beta;\gamma]} = \mathcal{R}_{ABCD}\mathcal{Y}^A_{,\alpha}\mathcal{N}^B\mathcal{Y}^C_{,\beta}\mathcal{Y}^D_{,\gamma} \tag{3}$$

where $\mathcal{R}_{ABCD}$ ($\bar{R}_{\mu\nu\alpha\beta}$) is the Riemann curvature of bulk (brane) manifold and

$$\bar{K}_{\alpha\beta} = \mathcal{G}_{AB}\left(\mathcal{Y}^A_{,\alpha\beta} + \Gamma^A_{CD}\mathcal{Y}^C_{,\alpha}\mathcal{Y}^D_{,\beta}\right)\mathcal{N}^B$$

is the extrinsic curvature.

At this stage we impose the cylinder condition, namely $\bar{g}_{\mu\nu,4} = 0$. This means that the fields $\bar{g}^{(0)}_{ij}(x)$, $A_\mu(x)$ and $\phi(x)$ are independent of the fifth coordinate x^4. In a sufficiently small neighborhood of $\overline{V}_5$, a local coordinate of the bulk space can be written as [6]

$$\mathcal{Z}^A = \mathcal{Y}^A + s\mathcal{N}^A \tag{4}$$

where s is a small parameter along $\mathcal{N}^A$ that parameterizes the large non compact extra dimension. In the adapted coordinate (4) the components of the bulk space metric can be rewritten as

$$\gamma_{\alpha\beta} = \mathcal{G}_{AB}\mathcal{Z}^A_{,\alpha}\mathcal{Z}^B_{,\beta} \quad \gamma_{\alpha 5} = \mathcal{G}_{AB}\mathcal{Z}^A_{,\alpha}\mathcal{N}^B \quad \gamma_{55} = \mathcal{G}_{AB}\mathcal{N}^A\mathcal{N}^B.$$

If we set $\mathcal{N}^A = \delta^A_5$, the metric of the bulk space can then be written in the form

$$\mathcal{G}_{AB} = \begin{pmatrix} g_{\alpha\beta} & 0 \\ 0 & \epsilon \end{pmatrix} \tag{5}$$

where in terms of the original brane metric and the corresponding extrinsic curvature, $g_{\alpha\beta}$ are given by

$$g_{\alpha\beta} = \bar{g}_{\alpha\beta} - 2s\bar{K}_{\alpha\beta} + s^2\bar{g}^{\mu\nu}\bar{K}_{\alpha\mu}\bar{K}_{\beta\nu}. \tag{6}$$

It should be clear that each $s =$ const. represents a (perturbed) brane that is isometrically embedded in a bulk for which the integrability conditions (2) read

$$R_{\mu\nu\alpha\beta} = 2K_{\mu[\beta}K_{\alpha]\nu} + \mathcal{R}_{ABCD}\mathcal{Z}^A_{,\mu}\mathcal{Z}^B_{,\nu}\mathcal{Z}^C_{,\alpha}\mathcal{Z}^D_{,\beta} \tag{7}$$

$$K_{\alpha[\beta;\gamma]} = \mathcal{R}_{ABCD}\mathcal{Z}^A_{,\alpha}\mathcal{N}^B\mathcal{Z}^C_{,\beta}\mathcal{Z}^D_{,\gamma} \tag{8}$$

with the extrinsic curvature now given by

$$K_{\alpha\beta} = -\frac{1}{2}\frac{\partial g_{\alpha\beta}}{\partial s} = \bar{K}_{\alpha\beta} - s\bar{g}^{\mu\nu}\bar{K}_{\mu\alpha}\bar{K}_{\nu\beta}. \tag{9}$$

At this point, it is desirable to calculate the Ricci scalar and Ricci tensor corresponding to our perturbed brane as this would enable us to define an effective action describing the theory. To this end, contraction of equation (7) leads to

$$R_{\alpha\beta} = 2\epsilon g^{\mu\nu} K_{\mu[\beta} K_{\nu]\alpha} + g^{\mu\nu} \mathcal{R}_{ABCD} \mathcal{Z}^A_{,\mu} \mathcal{Z}^C_{,\nu} \mathcal{Z}^B_{,\alpha} \mathcal{Z}^D_{,\beta}. \tag{10}$$

Using the identity

$$g^{\mu\nu} \mathcal{Z}^A_{,\mu} \mathcal{Z}^B_{,\nu} = \mathcal{G}^{AB} - \epsilon \mathcal{N}^A \mathcal{N}^B$$

the Ricci tensor and Ricci scalar of the perturbed brane are given by

$$R_{\alpha\beta} = \mathcal{R}_{AB} \mathcal{Z}^A_{,\alpha} \mathcal{Z}^B_{,\beta} - 2\epsilon g^{\mu\nu} K_{\mu[\nu} K_{\alpha]\beta} - \epsilon \mathcal{R}_{ABCD} \mathcal{Z}^B_{,\alpha} \mathcal{Z}^D_{,\beta} \mathcal{N}^A \mathcal{N}^C \tag{11}$$

and

$$R = \mathcal{R} + \epsilon K_{\mu\nu} K^{\mu\nu} - \epsilon K^2 - 2\epsilon \mathcal{R}_{AB} \mathcal{N}^A \mathcal{N}^B - \mathcal{R}_{ABCD} \mathcal{N}^A \mathcal{N}^B \mathcal{N}^C \mathcal{N}^D \tag{12}$$

where $K = g^{\mu\nu} K_{\mu\nu}$is the mean curvature. With a straightforward calculation in the Gaussian frame of the embedding represented by (5) we are led to the following equations

$$\mathcal{R}_{ABCD} \mathcal{Z}^B_{,\alpha} \mathcal{Z}^D_{,\beta} \mathcal{N}^A \mathcal{N}^C = -\epsilon K_{\alpha\mu} K^\mu_\beta - \epsilon K_{\alpha\beta,s} \tag{13}$$

$$\mathcal{R}_{AB} \mathcal{N}^A \mathcal{N}^B = \epsilon K_{\mu\nu} K^{\mu\nu} - \epsilon K_{,s} \tag{14}$$

$$\mathcal{R}_{ABCD} \mathcal{N}^A \mathcal{N}^B \mathcal{N}^C \mathcal{N}^D = 0 \tag{15}$$

so that the Ricci tensor and Ricci scalar of the perturbed brane take the following forms

$$R_{\alpha\beta} = \mathcal{R}_{AB} \mathcal{Z}^A_{,\alpha} \mathcal{Z}^B_{,\beta} + \epsilon \left(2K_{\alpha\mu} K^\mu_\beta - K K_{\alpha\beta} \right) + \epsilon K_{\alpha\beta,s} \tag{16}$$

and

$$R = \mathcal{R} - \epsilon \left(K^2 + K_{\mu\nu} K^{\mu\nu} \right) - 2\epsilon K_{,s}. \tag{17}$$

3 The model

Equation (17) suggests that the effective action for the brane-world geometry can now be written as

$$\mathcal{S}_{eff} = \frac{1}{2K^2_{(b)}} \int \sqrt{-g} R d^5x = \frac{1}{2K^2_{(b)}} \int \sqrt{-g} \left[\mathcal{R} - \epsilon \left(K^2 + K_{\mu\nu} K^{\mu\nu} \right) - 2\epsilon K_{,s} \right] d^5x \tag{18}$$

where $K_{(b)}$ is the gravitational constant of the brane world. In the spirit of the variational principle and following [6], we add an independent source as the Lagrangian of the confined matter ($\mathcal{L}_m$) together with the corresponding tension of the $5D$ brane (σ) to our geometrical Lagrangian in the form $\mathcal{L}_m - 2\sigma$. By using the variational principle or directly from equation (16) and (17), the field equations become

$$\begin{aligned} G_{\mu\nu} &= G_{AB} \mathcal{Z}^A_\mu \mathcal{Z}^B_\nu - \epsilon K K_{\mu\nu} + 2\epsilon K_{\mu\alpha} K^\alpha_\nu + \epsilon K_{\mu\nu,s} + \frac{\epsilon}{2} K^2 g_{\mu\nu} \\ &+ \frac{\epsilon}{2} K_{\alpha\beta} K^{\alpha\beta} g_{\mu\nu} - \epsilon K_{,s} g_{\mu\nu} + K^2_{(b)} S_{\mu\nu} \end{aligned} \tag{19}$$

where $S_{\mu\nu}$ is the matter energy-momentum tensor associated with the brane-world. This tensor consists of three parts

$$S_{ij} = \tau_{ij} - \sigma g^{(4)}_{ij} + A_i A_j \phi^2 \left(\mathcal{L}_m - \sigma\right) \tag{20}$$

$$S_{i4} = \phi^2 A_i \left(\mathcal{L}_m - \sigma\right) \tag{21}$$

$$S_{44} = \phi^2 \left(\mathcal{L}_m - \sigma\right) \tag{22}$$

with

$$\tau_{ij} = -\frac{2}{\sqrt{-g}} \frac{\delta(\mathcal{L}\sqrt{-g})}{\delta g^{ij}}.$$

In order to obtain the effective field equations on the brane, we have to replace the terms $\epsilon K_{,s}\, g_{\mu\nu}$ and $\epsilon K_{\mu\nu,s}$ in equation (19) with the $5D$ variables on the perturbed brane. Decomposing the $6D$ Riemann tensor as

$$\mathcal{R}_{ABCD} = \mathcal{C}_{ABCD} + \frac{1}{2}\left(\mathcal{G}_{A[C}\mathcal{R}_{D]B} - \mathcal{G}_{B[C}\mathcal{R}_{D]A}\right) - \frac{1}{10}\mathcal{G}_{A[C}\mathcal{G}_{D]B}\mathcal{R} \tag{23}$$

where $\mathcal{C}_{ABCD}$ is the $6D$ Weyl curvature, we find

$$\begin{aligned} K_{\mu\nu,s} - \frac{1}{5} g_{\mu\nu} K_{,s} \quad &= \quad -\frac{4}{3}\mathcal{E}_{\mu\nu} - \frac{2}{3} K_{\mu\alpha} K^{\alpha}_{\nu} - \frac{1}{5} K_{\alpha\beta} K^{\alpha\beta} g_{\mu\nu} - \frac{\epsilon}{3} R_{\mu\nu} + \frac{\epsilon}{15} R g_{\mu\nu} \\ &- \quad \frac{1}{3} K K_{\mu\nu} + \frac{1}{15} K^2 g_{\mu\nu} \end{aligned} \tag{24}$$

where $\mathcal{E}_{\mu\nu} = \mathcal{C}_{ABCD}\mathcal{N}^A\mathcal{N}^C\mathcal{Z}^B_{,\mu}\mathcal{Z}^D_{,\nu}$ is the electric part of the Weyl tensor. Since $\mathcal{E}_{\mu\nu}$ is traceless, we can not fix $K_{\mu\nu,s}$ by equation (24). To address this problem, we assume that the field equations on the bulk represent either a de-Sitter or anti de-Sitter space with a cosmological constant denoted by $\Lambda^{(B)}$, namely

$$G_{AB} = K^2_{(B)} \left(-\Lambda^{(B)}\mathcal{G}_{AB} + T^{(b)}_{AB}\right) \tag{25}$$

where

$$T^{(b)}_{AB} = \delta^{\mu}_{A}\delta^{\nu}_{B} S_{\mu\nu}\delta(s). \tag{26}$$

where $K_{(B)}$ is the gravitational constant of the bulk space. Now, taking the trace of equation (25), we find

$$\mathcal{R} = 3\Lambda^{(B)} K^2_{(B)}. \tag{27}$$

Using equations (17), (27) and (24) the field equations (19) can be rewritten as

$$G_{\mu\nu} = K^2_{(b)} S_{\mu\nu} - \frac{3}{4}\Lambda^{(B)} K^2_{(B)} g_{\mu\nu} - \epsilon K K_{\mu\nu} + \epsilon K_{\mu\alpha} K^{\alpha}_{\nu} - \frac{1}{2}\epsilon \left(K_{\mu\nu}K^{\mu\nu} - K^2\right) g_{\mu\nu}. \tag{28}$$

Equation (28) represents the field equations on the brane written in terms of the extrinsic curvature. However, the form of the field equations in our $4D$ world would also be of much interest and would tell us about the character of the fields A_i and ϕ in terms of which the metric (1) was introduced. To achieve this, we invoke Israel junction conditions. These junction conditions are obtained by assuming

Z_2 symmetry and substituting equation (25) into (19) and integrating the resulting equation in the normal direction to the brane, with the result

$$K_{\mu\nu} = -\frac{1}{2}\epsilon K^2_{(B)}\left[S_{\mu\nu} - \frac{1}{4}Sg_{\mu\nu}\right] \tag{29}$$

where S is the trace of $S_{\mu\nu}$.

To calculate the $4D$ field equations we first need to calculate the $4D$ part of the Ricci tensor and Ricci scalar of the $5D$ spacetime in the $\{i, 4\}$ coordinate using metric (1). The results are the following equations [7]

$$\begin{aligned}
R_{44} &= \frac{1}{4}\phi^4 F_{\mu\nu}F^{\mu\nu} - \phi\nabla^2\phi && (30)\\
R_{i4} &= \frac{1}{2}\phi^2\nabla^i F_{ij} + \frac{3}{2}\phi F_{ij}\nabla^j\phi + A_i R_{44} && (31)\\
R_{ij} &= R^{(4)}_{ij} - \frac{1}{2}\phi^2 F^m_i F_{mj} - \frac{1}{\phi}\nabla_i\nabla_j\phi + A_iA_jR_{44} + \frac{1}{2}\phi A_i\left(\phi\nabla^l F_{jl} + 3F_{jl}\nabla^l\phi\right) &&\\
&+ \frac{1}{2}\phi A_j\left(\phi\nabla^l F_{il} + 3F_{il}\nabla^l\phi\right) && (32)\\
R &= R^{(4)} - \frac{1}{4}\phi^2 F_{ij}F^{ij} - \frac{2}{\phi}\nabla^2\phi && (33)
\end{aligned}$$

where $R^{(4)}_{ij}$ and $R^{(4)}$ are Ricci tensor and Ricci scalar of $4D$ submanifold respectively with $F_{\mu\nu}$ being the usual electromagnetic field tensor. Defining the $4D$ energy-momentum tensor and tension as [8]

$$\begin{aligned}
\tau^{(4)}_{ij} &= \frac{1}{\phi}\tau_{ij}\\
\sigma^{(4)} &= \frac{1}{\phi}\sigma
\end{aligned} \tag{34}$$

and using the junction condition (29) together with equations (30-33) and equation (25) result in

$$\begin{aligned}
G^{(4)}_{ij} &= K^2_{(4)}\tau^{(4)}_{ij} + \phi^2\Sigma_{ij} + \frac{1}{\phi^2}\Theta_{ij} - \frac{\epsilon}{4}K^4_{(B)}\phi^2\Pi_{ij} && (35)\\
6\nabla^2\phi - 2M^2_H\phi - \lambda\phi^3 - \frac{3}{2}\phi^3 F_{ij}F^{ij} &+ 2K^2_{(4)}\phi\left(2\mathcal{L}^{(4)}_m - \tau^{(4)}\right) &&\\
&+\frac{3}{8}\epsilon K^4_{(B)}\phi^3\left(\tau^{(4)} - 3\mathcal{L}^{(4)}_m\right)\mathcal{L}^{(4)}_m = 0 && (36)\\
\nabla^i F_{ij} &+ \frac{3}{\phi}F_{ij}\nabla^j\phi = 0 && (37)
\end{aligned}$$

where

$$\Sigma_{ij} = \frac{1}{2}\left(F^l_i F_{lj} - \frac{1}{4}F^{mn}F_{mn}g^{(4)}_{ij}\right) \tag{38}$$

is the energy-momentum tensor of the Maxwell field and

$$\Theta_{ij} = \phi\nabla_i\nabla_j\phi - \phi\nabla^2\phi g^{(4)}_{ij} + \left(\frac{1}{2}M^2_H\phi^2 + \frac{\lambda}{4}\phi^4\right)g^{(4)}_{ij} \tag{39}$$

is the corresponding equation for the Higgs field with the local quadratic energy momentum correction being given by

$$
\begin{aligned}
\Pi_{ij} &= \frac{1}{16}\left[\mathcal{L}_m^{(4)}\tau^{(4)}g_{ij}^{(4)} - \mathcal{L}_m^{(4)}\tau_{ij}^{(4)} - \frac{3}{2}\left(\mathcal{L}_m^{(4)}\right)^2 g_{ij}^{(4)} - \tau^{(4)}\tau_{ij}^{(4)} + \frac{1}{2}\left(\tau^{(4)}\right)^2 g_{ij}^{(4)} + 4\tau_{im}^{(4)}\tau_j^{(4)m}\right. \\
&- \left. 2\tau^{(4)mn}\tau_{mn}^{(4)}g_{ij}^{(4)}\right] \qquad (40)
\end{aligned}
$$

Also the $4D$ gravitational constant, the Higgs mass, the coupling constant and the effective cosmological constant are respectively given by

$$
K_{(4)}^2 = \phi\left[K_{(b)}^2 - \frac{3\epsilon}{8}K_{(B)}^4\sigma\right] \qquad (41)
$$

$$
M_H^2 = -2\left[\frac{3}{4}\Lambda^{(B)}K_{(B)}^2 + \sigma K_{(b)}^2\right] \qquad (42)
$$

$$
\lambda = \frac{3\epsilon}{8}\,K_{(B)}^4\left(\sigma^{(4)}\right)^2 \qquad (43)
$$

$$
\Lambda^{(4)} = \frac{1}{2}M_H^2 - \frac{1}{4}\lambda\phi^2. \qquad (44)
$$

All these quantities have to be evaluated in the limit $s = 0^+$. From quantum field theory we know that the coupling constant of the Higgs field must be positive, hence $\epsilon = 1$, *i.e.* the large extra dimension must be spacelike. The present value of the Higgs field is of order $\phi \sim 10^{25}\text{cm}^{-1} \sim \frac{1}{r}$, where r is the radius of the compact dimension. This means that $r \sim 10^{-25}$cm which is compatible with the cylinder condition. As can be seen from equation (41), the gravitational constant of the $4D$ spacetime $K_{(4)}$ is expressed in terms of the gravitational constants of the brane and the bulk, the latter determining the coupling constant λ through equation (43). Now, if the bulk space did not exist, the coupling constant λ would disappear and the $4D$ cosmological constant $\Lambda^{(4)}$ would not agree with the present observations as it is solely expressed in terms of the Higgs mass, equation (44). However, in the presence of the bulk space, the $4D$ cosmological constant is modified by the addition of the term $\lambda\phi^2$ which would modify the value of $\Lambda^{(4)}$ in such a way as to make it more in line with the present observations. It is worth noticing at this point that the absence of the large extra dimension would render the theory ineffective in that $\Lambda^{(4)}$ cannot be modified and would retain a large value proportional to the Higgs mass, contrary to the present observations. As can be seen from equation (41), the existence of the small extra dimension is the cause of the variations in our $4D$ gravitational constant. We also note that as equation (35) shows, the local quadratic energy-momentum corrections are now multiplied by a factor ϕ^2, making the magnitude of these corrections dependent on the scale of the small extra dimension.

4 Localization of the gauge fields

The localization of the gauge fields [9] implies that gauge interactions should be confined to the brane. Assuming that the gauge interactions are concomitant with the quantum fluctuations of the geometry, it follows that equations (7) and (8) must also be compatible with the confinement of the gauge fields. In the Gauss-Codazzi equations, the basic variables are $g_{\alpha\beta}$ and $K_{\alpha\beta}$ and these quantities are influenced by the perturbations according to equations (6) and (9) respectively. From equation (6) we have

$$
g_{i4} = A_i'\phi'^2 = \bar{g}_{i4} - 2s\bar{K}_{i4} + s^2\bar{g}^{\mu\nu}\bar{K}_{i\mu}\bar{K}_{4\nu}
$$

$$g_{44} = \phi'^2 = \bar{g}_{44} - 2s\bar{K}_{44} + s^2\bar{g}^{\mu\nu}\bar{K}_{4\mu}\bar{K}_{4\nu} \tag{45}$$

which represents the variations of various components of our $5D$ brane metric. Also from the Junction conditions (29) we have

$$\bar{K}_{i4} = A_i\bar{K}_{44}. \tag{46}$$

Now from equations (45) and (46) one can see that

$$A_i' = A_i \tag{47}$$

implying that A_i's are localized in the sense that they remain unchanged independently of the quantum fluctuations of the brane-world. One should note that equation (47) could not be derived if the junction conditions were not assumed to hold, that is, the junction conditions are closely related to the localization of the gauge fields.

5 Conclusions

In this paper, we have studied the question of the unification of the Higgs and Maxwell fields in a model where a brane with a compact dimension is embedded in a bulk space having a constant curvature. Such a setup with its extra small and large dimensions enabled us to construct the Higgs and gauge fields purely in terms of the geometric quantities. We have also noted that within the context of the present model, there are three scales of energy; the unification of the fundamental forces in the bulk, the unification of the gauge fields on the brane arising from having an extra compact dimension and the energy scale of gravity in four dimensions. To account for the gauge fields other than Maxwell's, one would, in principle, increase the number of the compact dimensions of the brane in a straightforward manner.

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